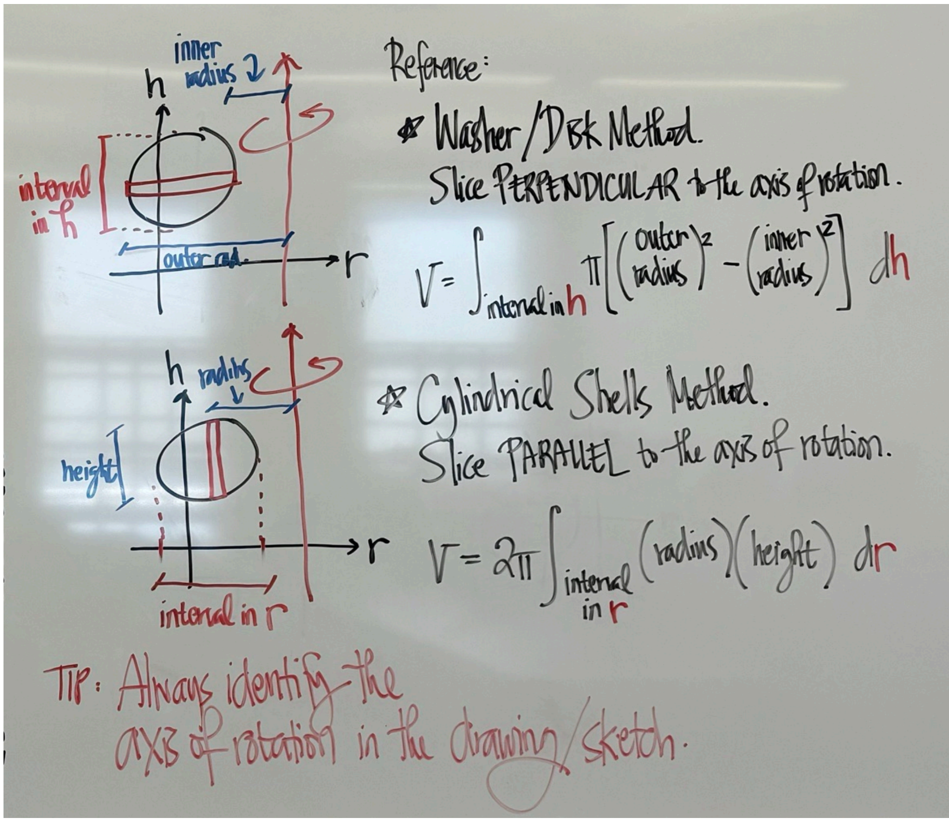
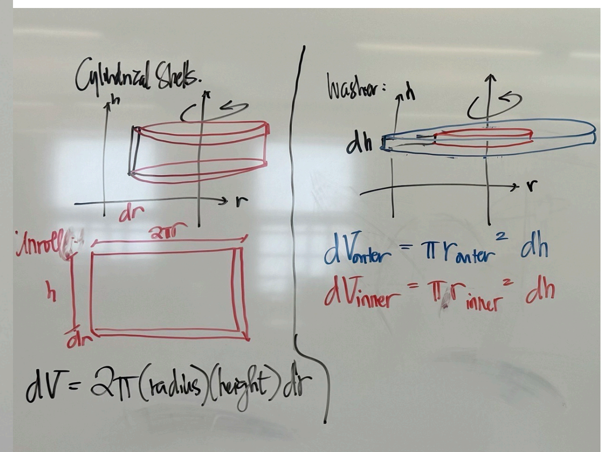
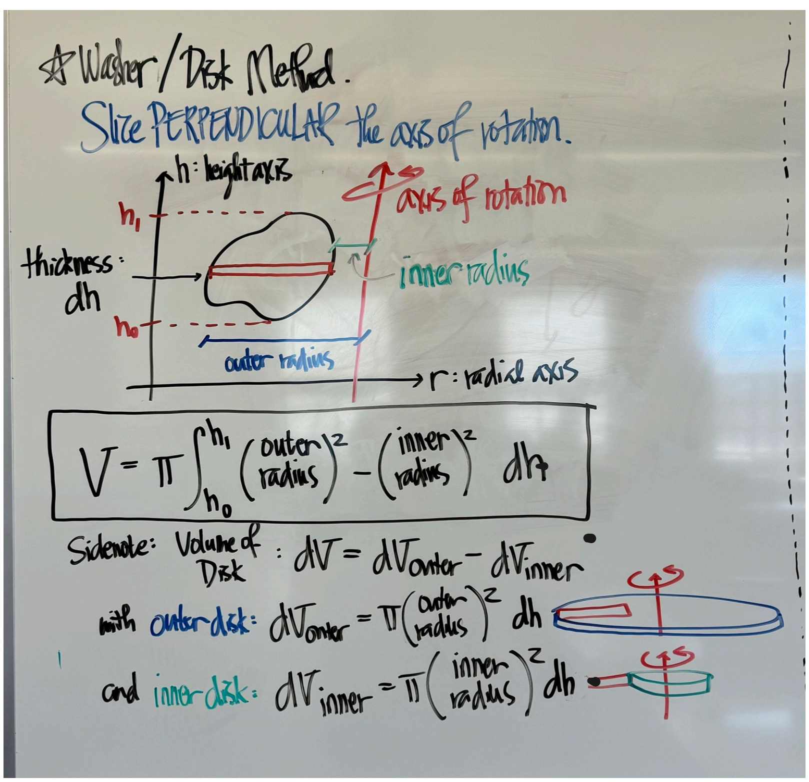
Reference for the Washer / Disk Method + Cylindrical Shells Method

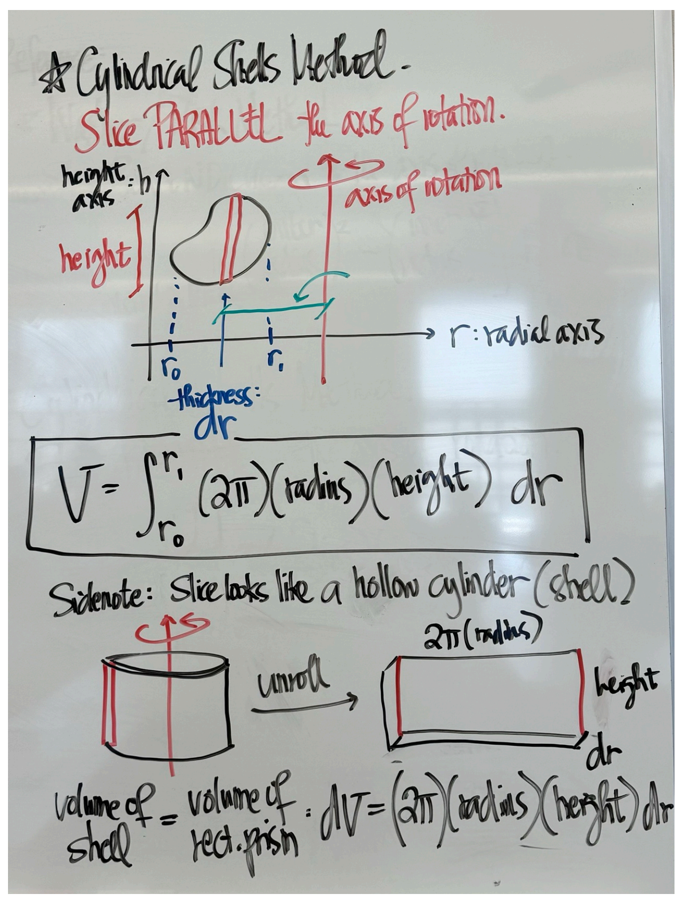
10AM Section.





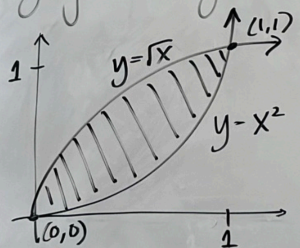
12PM + 2PM Section.





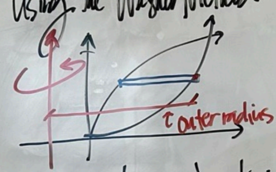
Example.

Ex. Let R be the region bounded by $y = x^2$ and $y = \sqrt{x}$;



Ⓐ Rotate R about the line $x = -1$.

Using the Washer Method:



thickness: dy, bounds: $y \in [0,1]$

outer radius: $r_{outer} = x + 1$ with $y = x^2$, $x = \sqrt{y}$

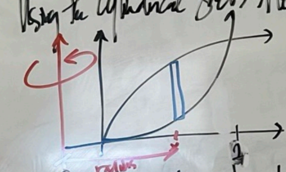
$r_{outer} = \sqrt{y} + 1$

inner radius: $r_{inner} = x + 1$ with $y = \sqrt{x}$, $x = y^2$

$= y^2 + 1$

$$V = \pi \int_0^1 (\sqrt{y}+1)^2 - (y^2+1)^2 \, dy$$

Using the Cylindrical Shells Method.



thickness: dx, bounds: $x \in [0,1]$

radius: $r = x + 1$

height: $h = y_{high} - y_{low}$

with y_{high}: $y = \sqrt{x}$, y_{low}: $y = x^2$

$h = \sqrt{x} - x^2$

$$V = \int_0^1 2\pi (x+1)(\sqrt{x} - x^2) \, dx$$

Ⓑ Rotate R about the line $y = 3$.

Using Washer Method:

thickness: dx, bounds: $x \in [0,1]$

outer rad: $r_{outer} = 3 - y$ with $y = x^2$

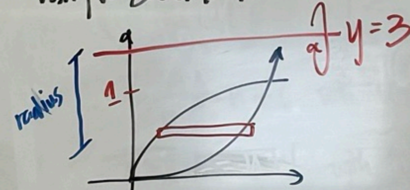
$= 3 - x^2$

inner radius: $r_{inner} = 3 - y$ with $y = \sqrt{x}$

$= 3 - \sqrt{x}$

$$V = \pi \int_0^1 (3 - x^2)^2 - (3 - \sqrt{x})^2 \, dx$$

Using the Shells Method.



thickness: dy, bounds: $y \in [0,1]$

radius: $r = 3 - y$

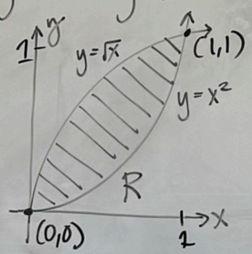
height: $h = x_{right} - x_{left}$ with $\begin{cases} x_{right}: y = x^2, \ x = \sqrt{y} \\ x_{left}: y = \sqrt{x}, \ x = y^2 \end{cases}$

$= \sqrt{y} - y^2$

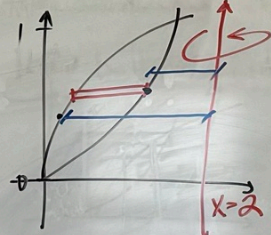
$$V = \int_0^1 2\pi (3-y)(\sqrt{y} - y^2) \, dy$$

Example.

ex. Let R be the region bounded by $y=x^2$ and $y=\sqrt{x}$



(a1) Rotate R about the line $x=2$. Use Washer Method.



thickness: dy

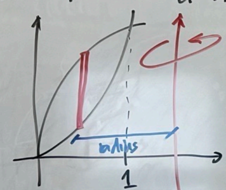
bounds: $y \in [0,1]$

$r_{outer} = 2-x$ with $y=\sqrt{x}$, $x=y^2$; $r_{outer} = 2-y^2$

$r_{inner} = 2-x$ with $y=x^2$, $x=\pm\sqrt{y}$; $r_{inner} = 2-\sqrt{y}$

$$V = \int_0^1 \pi\left[(2-y^2)^2 - (2-\sqrt{y})^2\right] dy$$

(a2) Rotate R about $x=2$. Use Cylindrical Shells.



thickness: dx

bounds: $x \in [0,1]$

radius: $2-x$

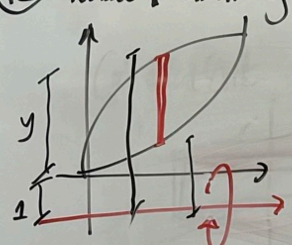
height: $y_{high} - y_{low}$

with y_{high}: $y=\sqrt{x}$

y_{low}: $y=x^2$

$$V = \int_0^1 2\pi(2-x)\left(\sqrt{x}-x^2\right) dx$$

(b1) Rotate R about $y=-1$. Use Washer.



thickness: dx

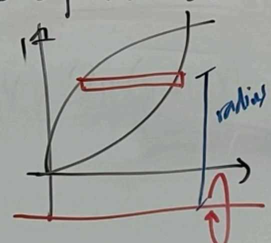
bounds: $x \in [0,1]$

outer radius: $r_{outer} = y+1$ with $y=\sqrt{x}$; $r_{outer} = \sqrt{x}+1$

inner rad: $r_{inner} = y+1$ with $y=x^2$; $r_{inner} = x^2+1$

$$V = \int_0^1 \pi\left[(\sqrt{x}+1)^2 - (x^2+1)^2\right] dx$$

(b2) Rotate R about $y=-1$.
Use Cylindrical Shells.



thickness: dy

bounds: $y \in [0,1]$

radius: $y+1$

height: $x_{right} - x_{left}$ with $\begin{cases} x_{right}: y=x^2,\ x=\pm\sqrt{y},\ x=\sqrt{y} \\ x_{left}: y=\sqrt{x},\ x=y^2 \end{cases}$

$=\sqrt{y}-y^2$

$$\boxed{V = \int_0^1 2\pi(y+1)\left(\sqrt{y}-y^2\right) dy}$$