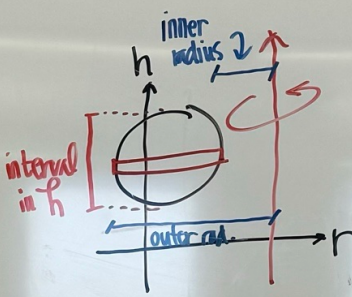


Reference for the Washer/Disk Method + Cylindrical Shells Method

WAM Section.

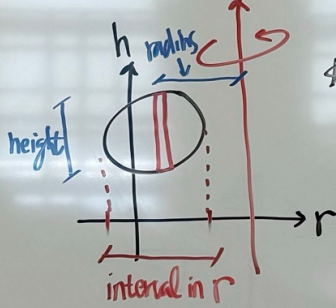
Reference:

★ Washer/Disk Method.
Slice PERPENDICULAR to the axis of rotation.



$$V = \int_{\text{interval in } h} \pi \left[(\text{outer radius})^2 - (\text{inner radius})^2 \right] dh$$

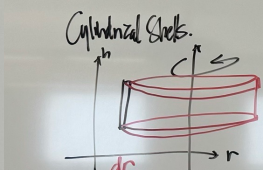
★ Cylindrical Shells Method.
Slice PARALLEL to the axis of rotation.



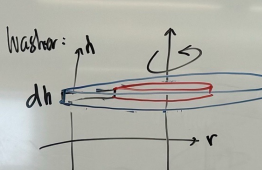
$$V = 2\pi \int_{\text{interval in } r} (\text{radius})(\text{height}) dr$$

TIP: Always identify the axis of rotation in the drawing/sketch.

Cylindrical Shells:



Washer:



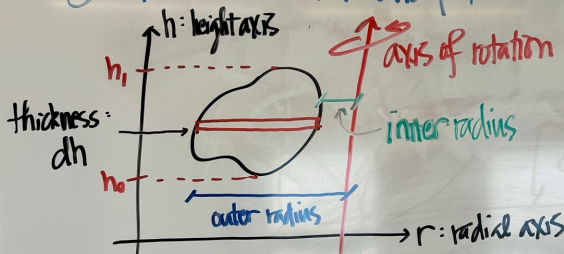
$$dV_{\text{washer}} = \pi R^2 dh$$

$$dV_{\text{inner}} = \pi r^2 dh$$

$$dV = 2\pi (\text{radius})(\text{height}) dr$$

12PM + 2PM Section.

★ Washer/Disk Method.
Slice PERPENDICULAR the axis of rotation.

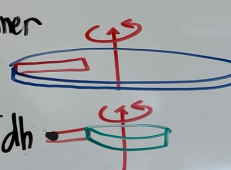


$$V = \pi \int_{h_0}^{h_1} (\text{outer radius})^2 - (\text{inner radius})^2 dh$$

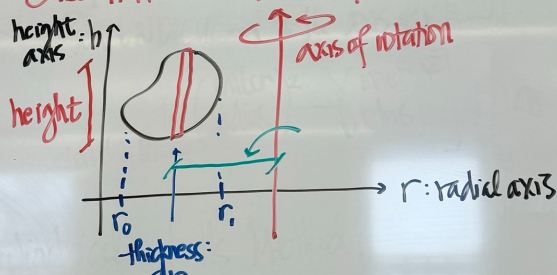
Sidenote: Volume of Disk: $dV = dV_{\text{outer}} - dV_{\text{inner}}$

with outer disk: $dV_{\text{outer}} = \pi (\text{outer radius})^2 dh$

and inner disk: $dV_{\text{inner}} = \pi (\text{inner radius})^2 dh$

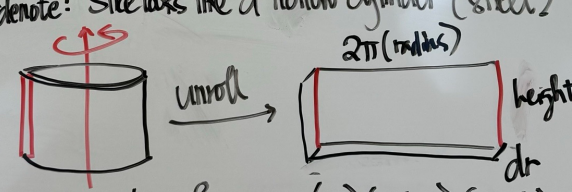


★ Cylindrical Shells Method.
Slice PARALLEL the axis of rotation.



$$V = \int_{r_0}^{r_1} (2\pi)(\text{radius})(\text{height}) dr$$

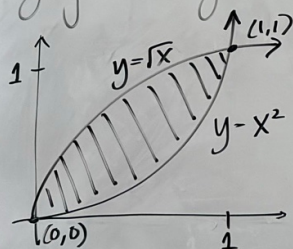
Sidenote: Slice looks like a hollow cylinder (shell)



volume of shell = volume of rect. prism: $dV = (2\pi)(\text{radius})(\text{height}) dr$

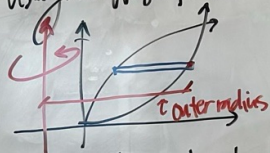
Example.

Ex Let R be the region bounded by $y = x^2$ and $y = \sqrt{x}$;



A) Rotate R about the line $x = -1$.

Using the Washer Method:



thickness: dy , bounds: $y \in [0, 1]$

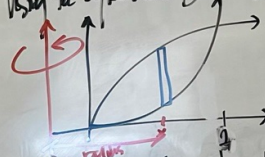
outer radius: $r_{\text{outer}} = x + 1$ with $y = x^2$, $x = \sqrt{y}$

$r_{\text{outer}} = \sqrt{y} + 1$

inner radius: $r_{\text{inner}} = x + 1$ with $y = \sqrt{x}$, $x = y^2$
 $= y^2 + 1$

$$V = \pi \int_0^1 (\sqrt{y} + 1)^2 - (y^2 + 1)^2 dy$$

Using the Cylindrical Shells Method.



thickness: dx , bounds: $x \in [0, 1]$

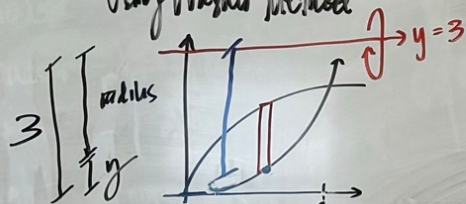
radius: $r = x + 1$

height: $h = y_{\text{upper}} - y_{\text{lower}}$
with $y_{\text{upper}} = y = \sqrt{x}$, $y_{\text{lower}} = y = x^2$
 $h = \sqrt{x} - x^2$

$$V = \int_0^1 2\pi (x+1)(\sqrt{x} - x^2) dx$$

B) Rotate R about the line $y = 3$.

Using Washer Method:



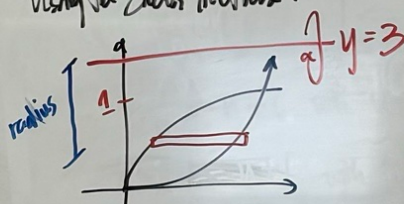
thickness: dx , bounds: $x \in [0, 1]$

outer rad: $r_{\text{outer}} = 3 - y$ with $y = x^2$
 $= 3 - x^2$

inner radius: $r_{\text{inner}} = 3 - y$ with $y = \sqrt{x}$
 $= 3 - \sqrt{x}$

$$V = \pi \int_0^1 (3 - x^2)^2 - (3 - \sqrt{x})^2 dx$$

Using the Shells Method.



thickness: dy , bounds: $y \in [0, 1]$

radius: $r = 3 - y$

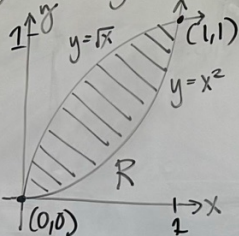
height: $h = x_{\text{right}} - x_{\text{left}}$ with $\begin{cases} x_{\text{right}} = y = x^2, x = \sqrt{y} \\ x_{\text{left}} = y = \sqrt{x}, x = y^2 \end{cases}$
 $= \sqrt{y} - y^2$

$$V = \int_0^1 2\pi (3 - y)(\sqrt{y} - y^2) dy$$

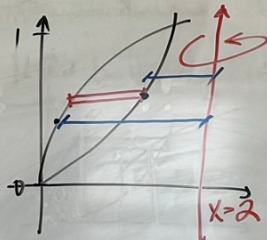
Example.

ex. let R be the region bounded by

$$y = x^2 \text{ and } y = \sqrt{x}$$



(a1) Rotate R about the line $x=2$. Use Washer Method.



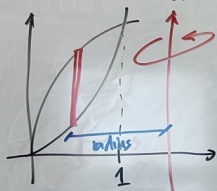
thickness: dy
bounds: $y \in [0,1]$

$$r_{\text{outer}} = 2 - x \text{ with } y = \sqrt{x}, x = y^2; r_{\text{outer}} = 2 - y^2$$

$$r_{\text{inner}} = 2 - x \text{ with } y = x^2, x = \pm\sqrt{y}; r_{\text{inner}} = 2 - \sqrt{y}$$

$$V = \int_0^1 \pi \left[(2 - y^2)^2 - (2 - \sqrt{y})^2 \right] dy$$

(a2) Rotate R about $x=2$. Use Cylindrical Shells.



thickness: dx

bounds: $x \in [0,1]$

radius: $2 - x$

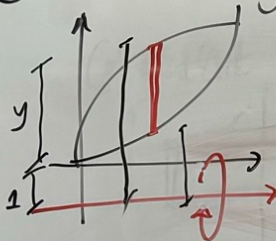
height: $y_{\text{upper}} - y_{\text{lower}}$

with $y_{\text{upper}}: y = \sqrt{x}$

$y_{\text{lower}}: y = x^2$

$$V = \int_0^1 2\pi(2-x)(\sqrt{x} - x^2) dx$$

(b1) Rotate R about $y=-1$. Use Washer.



thickness: dx

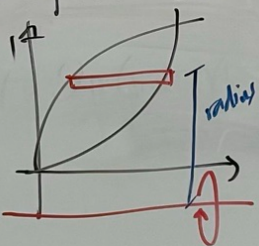
bounds: $x \in [0,1]$

outer radius: $r_{\text{outer}} = y + 1$ with $y = \sqrt{x}; r_{\text{outer}} = \sqrt{x} + 1$

inner rad: $r_{\text{inner}} = y + 1$ with $y = x^2; r_{\text{inner}} = x^2 + 1$

$$V = \int_0^1 \pi \left[(\sqrt{x} + 1)^2 - (x^2 + 1)^2 \right] dx$$

(b2) Rotate R about $y=-1$.
Use Cylindrical Shells.



thickness: dy

bounds: $y \in [0,1]$

radius: $y + 1$

height: $x_{\text{right}} - x_{\text{left}}$ with

$$\begin{cases} x_{\text{right}}: y = x^2, x = \pm\sqrt{y}, x = \sqrt{y} \\ x_{\text{left}}: y = \sqrt{x}, x = y^2 \end{cases}$$

$$= \sqrt{y} - y^2 \quad V = \int_0^1 2\pi(y+1)(\sqrt{y} - y^2) dy$$

